Vacuum fluctuations of twisted fields in the spacetime of a cosmic string-thermal effects

1993 J. Phys. A: Math. Gen. 26 L777

(http://iopscience.iop.org/0305-4470/26/17/004)

J. Phys. A: Math. Gen. 26 (1993) L777–L780. Printed in the UK

LETTER TO THE EDITOR

Vacuum fluctuations of twisted fields in the spacetime of a cosmic string—thermal effects

Marek Rogatko

Institute of Physics, University of M Curie-Skłodowska, 20-105 Lublin, pl. M. Curie Skłodowskiej 1, Poland

Received 3 June 1993

Abstract. In the case of quantum field theory at finite temperature we investigate the thermal Euclidean Green function for massive twisted scalar fields in the spacetime of an infinite straight cosmic string. Then taking the limit, when mass goes to zero, we obtain the integral form of the mean-square field as a function of temperature.

Grand unified cosmic strings have been treated with great interest during the past few years. They have astrophysical importance because loops of string provide the seeds for the process of galaxies formation. They could also act as gravitational lenses, doubling the images of quasars. After the detection of anisotropies in the cosmic microwave background radiation it seemed that these anisotropies may be caused by fast-moving long strings [1].

The spacetime generated by an infinite straight cosmic string is conical in the outside region and the metric is given by [2]

$$ds^2 = -dt^2 + dr^2 + \frac{r^2}{p^2} d\theta^2 + dz^2 \tag{1}$$

where $p^{-1} = 1 - 4G\mu < 1$. We redefine the coordinate $\tilde{\theta} = \theta/p$, so that $\tilde{\theta}$ takes values from zero to $2\pi/p$.

It was pointed out [3] that, in addition to the existing standard type of fields, which can be regarded as the cross-section of a trivial bundle, there should also be so-called twisted fields corresponding to the cross-section of a non-trivial vector bundle. Several works have been devoted to the problem of twisted scalar fields in various kinds of spacetimes [4].

The aim of this letter is to determine the Euclidean thermal Green function for massive twisted scalar fields in the spacetime of a cosmic string. Setting the mass equal to zero, we obtain the thermal average.

Several authors have dealt with the problem of two-point functions [5]. In our consideration we apply the method used in [6, 7]. By this prescription we acquire the following Green function for twisted scalar fields conformally coupled to gravitation:

$$G_p(x, x') = \langle 0|\phi(x)\phi(x')|0\rangle$$
$$= \frac{p}{4\pi^2} \sum_{m=-\infty}^{\infty} e^{i(m+1/2)(\theta-\theta')} \times \int_0^\infty dK\, K J_{p|m+1/2|}(Kr') J_{p|m+1/2|}(Kr) K_0[(K^2+M^2)^{1/2}\xi] \tag{2}$$

where $J_p(x)$ is a Bessel function, K_0 is a modified Bessel function, M stands for the mass of the scalar field, $\xi^2=(z-z')^2-(t-t')^2$. Using the integral representation of $K_0[(K^2+M^2)^{1/2}\xi]$ and performing integration with respect to K [8] we obtain

$$G_p(x,x')=\frac{p}{4\pi^2\xi^2}\sum_{m=-\infty}^{\infty}e^{i(m+1/2)(\theta-\theta')}\int_0^\infty dt\, I_{p|m+1/2|}\left(\frac{2rr'}{\xi^2}t\right)\exp\left(-\frac{r^2+r'^2+\xi^2}{\xi^2}t-\frac{M^2\xi^2}{4t}\right). \tag{3}$$

To attain the integral form of the above equation we have to use an integral representation of $I_\mu(x)$ [8]. After lengthy algebra one obtains the following expression, which will be necessary in our further considerations:

$$\sum_{m=-\infty}^{\infty}\exp(i(m+1/2)\phi-p|m+1/2[x)\sin(p\pi|m+1/2|)$$
$$=\cosh(px/2)\left[\frac{\sin\left(\frac{p\pi+\phi}{2}\right)}{\cosh(px)-\cos(p\pi+\phi)}+\frac{\sin\left(\frac{p\pi-\phi}{2}\right)}{\cosh(px)-\cos(p\pi-\phi)}\right] \tag{4}$$

where $\phi=\theta-\theta'$. For brevity, we denote expressions in brackets in equation (4), respectively, by $F(\phi_+)$ and $F(\phi_-)$. Making use of the following identity:

$$\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}e^{i(m+1/2)\psi}\cos(p|m+1/2|\sigma)=\frac{1}{2p}\left[\delta\left(\sigma-\frac{\psi}{p}\right)+\delta\left(\sigma+\frac{\psi}{p}\right)\right] \tag{5}$$

carrying out summation with respect to m, replacing t by $-i\tau$ and integrating we finally reach the form of Euclidean Green function:

$$G_{Ep}(x,x')=\frac{1}{4\pi^2}\frac{M}{d}K_1(Md)-\frac{p}{4\pi^3}\int_0^\infty dx\,\frac{M\cosh(px/2)}{D}K_1(MD)(F(\phi_+)+F(\phi_-)) \tag{6}$$

where

$$\tilde{\xi}^2=(z-z')^2+(\tau-\tau')^2$$
$$d=[r^2+r'^2-2rr'\cos(\phi/p)+\tilde{\xi}^2]^{1/2}$$
$$D=(r^2+r'^2+2rr'\cosh(x)+\tilde{\xi}^2)^{1/2}.$$

Having the Euclidean Green function in the form (6) and with the aid of the Laplace inverse transformation [9] we can determine the Euclidean heat kernel as follows:

$$K_E(x,x',s)=\frac{1}{16\pi^2s^2}\exp\left(-\frac{d^2}{4s}-M^2s\right)-\frac{p}{16\pi^3s^2}\int_0^\infty dx\cosh(px/2)[F(\phi_++F(\phi_-)]\exp\left(-\frac{D^2}{4s}-M^2s\right). \tag{7}$$

It turns out that for ultrastatic metrics [10] the thermal heat kernel may be factorized as follows:

$$K_{ET}=\theta_3\left(\frac{\mathrm{i}\beta(\tau-\tau')}{4s}\,\frac{\mathrm{i}\beta^2}{4\pi s}\right)K_E(x,x',s), \tag{8}$$

where $\beta=1/kT$, k is the Boltzmann constant, T denotes temperature and $\theta_3(x|\mu)$ is the theta function [11]. Using the formula in the Schwinger–DeWitt formalism, one can write the Euclidean thermal Green function in the form:

$$G_{ET}(x,x',s)=\int_0^\infty \mathrm{d}s\,K_{ET}(x,x',s). \tag{9}$$

Following the procedure presented by Linet [9], i.e. changing the variables of integration and setting $M=0$ to get 'a more practical form', we have

$$\begin{aligned} G_{ET}^{M=0}(x,x')&=\frac{1}{4\beta^2}\int_0^\infty \mathrm{d}t\,\theta_3\left(\frac{\mathrm{i}\pi^2}{\beta}(\tau-\tau')|\mathrm{i}\pi t\right)\exp\left(-\frac{\pi^2}{\beta^2}d^2t\right)\mathrm{d}t\\ &+\frac{p}{4\pi\beta^2}\int_0^\infty \mathrm{d}x\int_0^\infty \mathrm{d}t\,\theta_3\left(\frac{\mathrm{i}\pi^2}{\beta}(\tau-\tau')t|\mathrm{i}\pi t\right)\cosh(px/2)\\ &\times[F(\phi_+ + F(\phi_-)]\exp\left(-\frac{\pi^2}{\beta^2}D^2t\right). \end{aligned} \tag{10}$$

Evaluating the t-integral [10] one obtains

$$\begin{aligned} G_{ET}^{M=0}(x,x')&=\frac{\sinh\left(\frac{2\pi}{\beta}\delta\right)}{4\pi\beta\delta\left[\cosh\left(\frac{2\pi}{\beta}\delta\right)-\cos\left(\frac{2\pi}{\beta}(\tau-\tau')\right)\right]}\\ &+\frac{p}{4\pi^2\beta\Delta}\int_0^\infty \mathrm{d}x\,\frac{\cosh(px/2)\sinh\left(\frac{2\pi}{\beta}\Delta\right)}{\left[\cosh\left(\frac{2\pi}{\beta}\Delta\right)-\cos\left(\frac{2\pi}{\beta}(\tau-\tau')\right)\right]}[F(\phi_+)+F(\phi_-)] \end{aligned} \tag{11}$$

where

$$\delta^2=(z-z')^2+r^2+r'^2-2rr'\cos(\phi/p)$$

$$\Delta^2=(z-z')^2+r^2+r'^2+2rr'\cosh(x).$$

It is interesting to evaluate the thermal average

$$\langle\varphi^2(x)\rangle\beta=\lim_{x\to x'}\left[G_{ET}^{M=0}(x,x')-\frac{1}{4\pi^2d^2}\right]. \tag{12}$$

Taking into account equations (11) and (12) and the asymptotic behaviour of t-integration [9]

$$\frac{\sinh 2a}{a(\cosh a - \cos 2k)} \approx \frac{1}{k^2 + a^2} + 1/3$$

as $k, a \to 0$ we obtain

$$\langle \varphi^2(x) \rangle_\beta = 1/12\beta^2 - \frac{p \sin(p\pi/2)}{4\pi^2 \delta \beta} \int_0^\infty \mathrm{d}x \frac{\coth\left[\frac{2\pi}{\beta} \delta \cosh(x/2)\right] \cosh(px/2)}{\cosh(x/2)[\cosh(px) - \cos(p\pi)]}. \tag{13}$$

When the temperature goes to zero, i.e. $\beta \to \infty$ we have

$$\langle \varphi^2(x) \rangle_\infty = -\frac{p \sin(p\pi/2)}{8\pi^3 \delta^2} \int_0^\infty \mathrm{d}x \frac{\cosh(px/2)}{\cosh(x/2)^2[\cosh(px) - \cos(p\pi)]}. \tag{14}$$

On the contrary when $\beta \to 0$ the limit is

$$\langle \varphi^2(x) \rangle_0 \approx 1/12\beta^2 - \frac{p \sin(p\pi/2)}{4\pi^2 \delta \beta} \int_0^\infty \mathrm{d}x \frac{\cosh(px/2)}{\cosh(px/2)[\cosh(px) - \cos(p\pi)]}. \tag{15}$$

We obtain an integral form for the mean-square field as an integral function dependent on temperature. Besides the well known part $\langle \varphi^2(x) \rangle_\beta$, it consists of the integral part which is responsible for the geometry of the problem under consideration.

References

[1] Hara T, Mähönen P and Miyoshi S 1993 *Phys. Rev.* D **47** 2297
[2] Gott J R 1985 *Astrophys. J.* **288** 411
[3] Isham C J 1978 *Proc. R. Soc.* A **363** 383
Avis S J and Isham C 1978 *Proc. R. Soc.* A **363** 581.
[4] Ford L H 1980 *Phys. Rev.* D **21** 949
Banach R and Dowker J S 1979 *J. Phys. A: Math Gen.* **12** 2527
Banach R and Dowker J S 1979 *J. Phys. A: Math. Gen.* **12** 2545
Matsas G E A 1990 *Phys. Rev.* D **41** 3846
[5] Linet B 1987 *Phys. Rev.* D **35** 536
Frolov V P and Serebriany E M 1987 *Phys. Rev.* D **38** 1331
Heliwell T M and Konkowski D A 1986 *Phys. Rev.* D **34** 1918
[6] Smith A G 1989 *The Formation and Evolution of Cosmic Strings* ed G Gibbons, S W Hawking and T Vachaspati (Cambridge: Cambridge University Press)
[7] Shiraishi K and Hirenzaki S 1992 *Class. Quantum Grav.* **9** 2277
[8] Gradsteyn S and Ryzhik I M 1965 *Tables of Integrals, Series and Products* (New York: Academic)
[9] Linet B 1992 *Class. Quantum Grav.* **9** 2429
[10] Page D N 1982 *Phys. Rev.* D **25** 1028
[11] Mumford D 1983, 1984 Tata Lectures on Theta Function I, II (Boston: Basel Birkhäuser)